Paper Reference 9FM0/01
Pearson Edexcel
Level 3 GCE

Further Mathematics

Advanced
Paper 1: Core Pure Mathematics 1

Monday 3 June 2019 – Morning

Time: 1 hour 30 minutes plus your additional time allowance.

MATERIALS REQUIRED FOR EXAMINATION
Mathematical Formulae and Statistical Tables (Green)
Calculator

ITEMS INCLUDED WITH QUESTION PAPERS
Diagram Book
Answer Book

X61177A

Pearson

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Book and on the Diagram Book, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Book or on the separate diagrams – there may be more space than you need.

Do NOT write on the Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

There are 8 questions in this Question Paper.

The total mark for this paper is 75

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

Answer ALL questions.

Write your answers in the Answer Book.

1.

$$f(z) = z^4 + az^3 + bz^2 + cz + d$$

where a, b, c and d are real constants.

Given that $-1 + 2i$ and $3 - i$ are two roots of the equation $f(z) = 0$

(a) show all the roots of $f(z) = 0$ on a single Argand diagram,

(4 marks)

(b) find the values of a, b, c and d

(5 marks)

(Total for Question 1 is 9 marks)

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2. **Show that**

$$\int_0^{\infty} \frac{8x - 12}{(2x^2 + 3)(x + 1)}\,dx = \ln k$$

where k is a rational number to be found.

(Total for Question 2 is 7 marks)

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3. Look at the diagram for Question 3 in the Diagram Book.

It is NOT to scale.

It shows the design for a table top in the shape of a rectangle ABCD

The length of the table, AB, is 1·2 metres.

The area inside the closed curve is made of glass and the surrounding area, shown shaded in the diagram, is made of wood.

The perimeter of the glass is modelled by the curve with polar equation

$$r = 0{\cdot}4 + a\cos 2\theta \qquad 0 \le \theta < 2\pi$$

where a is a constant.

(a) Show that $a = 0{\cdot}2$

(2 marks)

(continued on the next page)

3. continued.

Hence, given that AD = 60 cm,

(b) find the area of the wooden part of the table top, giving your answer in m^2 to 3 significant figures.
(8 marks)

(Total for Question 3 is 10 marks)

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4. Prove that, for $n \in \mathbb{Z}$, $n \geq 0$

$$\sum_{r=0}^{n} \frac{1}{(r+1)(r+2)(r+3)} = \frac{(n+a)(n+b)}{c(n+2)(n+3)}$$

where a, b and c are integers to be found.

(Total for Question 4 is 5 marks)

5. A tank at a chemical plant has a capacity of 250 litres.
The tank initially contains 100 litres of pure water.

Salt water enters the tank at a rate of 3 litres every minute.
Each litre of salt water entering the tank contains 1 gram of salt.

It is assumed that the salt water mixes instantly with the contents of the tank upon entry.

At the instant when the salt water begins to enter the tank, a valve is opened at the bottom of the tank and the solution in the tank flows out at a rate of 2 litres per minute.

(continued on the next page)

5. continued.

Given that there are S grams of salt in the tank after t minutes,

(a) show that the situation can be modelled by the differential equation

$$\frac{dS}{dt} = 3 - \frac{2S}{100 + t}$$

(4 marks)

(b) Hence find the number of grams of salt in the tank after 10 minutes.
(5 marks)

When the concentration of salt in the tank reaches 0·9 grams per litre, the valve at the bottom of the tank must be closed.

(c) Find, to the nearest minute, when the valve would need to be closed.
(3 marks)

(d) Evaluate the model.
(1 mark)

(Total for Question 5 is 13 marks)

Turn over

6. Prove by induction that for all positive integers n

$$f(n) = 3^{2n + 4} - 2^{2n}$$

is divisible by 5

(Total for Question 6 is 6 marks)

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7. The line L_1 has equation

$$\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-4}{3}$$

The line L_2 has equation

$$\underline{r} = \underline{i} + 3\underline{k} + t(\underline{i} - \underline{j} + 2\underline{k})$$

where t is a scalar parameter.

(a) Show that L_1 and L_2 lie in the same plane.
(3 marks)

(b) Write down a vector equation for the plane containing L_1 and L_2
(1 mark)

(c) Find, to the nearest degree, the acute angle between L_1 and L_2
(3 marks)

(Total for Question 7 is 7 marks)

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8. **A scientist is studying the effect of introducing a population of white–clawed crayfish into a population of signal crayfish.**
At time t years, the number of white–clawed crayfish, w, and the number of signal crayfish, s, are modelled by the differential equations

$$\frac{dw}{dt} = \frac{5}{2}(w - s)$$

$$\frac{ds}{dt} = \frac{2}{5}w - 90e^{-t}$$

(a) **Show that**

$$2\frac{d^2w}{dt^2} - 5\frac{dw}{dt} + 2w = 450e^{-t}$$

(3 marks)

(b) **Find a general solution for the number of white–clawed crayfish at time t years.**
(6 marks)

(continued on the next page)

8. continued.

(c) Find a general solution for the number of signal crayfish at time t years.

(2 marks)

The model predicts that, at time T years, the population of white–clawed crayfish will have died out.

Given that $w = 65$ and $s = 85$ when $t = 0$

(d) find the value of T, giving your answer to 3 decimal places.

(6 marks)

(e) Suggest a limitation of the model.

(1 mark)

(Total for Question 8 is 18 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER
